

# Supernova-driven outflows and chemical evolution of dwarf spheroidal galaxies

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We present a general phenomenological model for the metallicity distribution (MD) in terms of  $[\text{Fe}/\text{H}]$  for dwarf spheroidal galaxies (dSphs). These galaxies appear to have stopped accreting gas from the intergalactic medium and are fossilized systems with their stars undergoing slow internal evolution. For a wide variety of infall histories of unprocessed baryonic matter to feed star formation, most of the observed MDs can be well described by our model. The key requirement is that the fraction of the gas mass lost by supernova-driven outflows is close to unity. This model also predicts a relationship between the total stellar mass and the mean metallicity for dSphs in accord with properties of their dark matter halos. The model further predicts as a natural consequence that the abundance ratios  $[\text{E}/\text{Fe}]$  for elements such as O, Mg, and Si decrease for stellar populations at the higher end of the  $[\text{Fe}/\text{H}]$  range in a dSph. We show that for infall rates far below the net rate of gas loss to star formation and outflows, the MD in our model is very sharply peaked at one  $[\text{Fe}/\text{H}]$  value, similar to what is observed in most globular clusters. This suggests that globular clusters may be end members of the same family as dSphs.

chemical evolution | dwarf spheroidal galaxies | supernovae

In this paper we show that supernova-driven gas outflows play a prominent role in the chemical evolution of dwarf spheroidal galaxies (dSphs). In the framework of hierarchical structure formation based on the cold dark matter cosmology, dwarf galaxies are the building blocks of large galaxies such as the Milky Way. In support of this picture, some recent observations showed that elemental abundances in dSphs of the Local Group match those in the Milky Way halo at low metallicities (e.g., [1, 2, 3, 4, 5]; see [6] for a review of earlier works). It is expected that detailed studies of chemical evolution of dwarf galaxies can shed important light on the formation and evolution of the Milky Way in particular and large galaxies in general. Here we present an analysis of the evolution of  $[\text{Fe}/\text{H}] = \log(\text{Fe}/\text{H}) - \log(\text{Fe}/\text{H})_\odot$  focussing on dSphs. The approach is a phenomenological one that takes into account infall of gas into the dark matter halos associated with these galaxies, star formation (SF) within the accumulated gas, and outflows driven by supernova (SN) explosions. The sources for production of Fe are core-collapse SNe (CCSNe) from progenitors of 8–100  $M_\odot$  and Type Ia SNe (SNe Ia) associated with stars of lower masses in binaries. Observations require that some SNe Ia must form early along with CCSNe without a significant delay. It will be shown that there is a direct and simple connection between the metallicity distribution (MD) for a given dSph and two parameters  $\lambda_{\text{Fe}}/\lambda$  and  $\alpha$ . The ratio refers to the net rate  $\lambda_{\text{Fe}}$  of Fe production and the net rate  $\lambda$  of gas loss to SF and SN-driven outflows, and  $\alpha$  indicates the promptness for reaching peak infall rates. This model explicitly predicts the ratio of the stellar mass in the dSph to the total mass of the host dark matter halo.

Phenomenological models for chemical evolution have a long history (e.g., [7]) and were applied to dSphs previously (e.g., [8, 9, 10]). Dynamic models for dSphs including dark matter were also studied (e.g., [11, 12]). The first effort was made in [13] to reconcile models of hierarchical structure for-

mation involving dark matter halos with the then-available luminosity-radius-metallicity relationships for dwarf galaxies. There it was shown that SN-driven outflows could explain the observed trends. Recent observations [14] give results for eight dSphs with rather detailed structure of their MDs and provide a basis for exploring models of their chemical evolution (e.g., [15]). Among the key issues that we try to address are MDs exhibited by stellar populations of dSphs. Our general approach follows that of Lynden-Bell described in his incisive and excellent article on “theories” of the chemical evolution of galaxies [16]. It will be shown that the metallicity at which the MD peaks is directly related to the efficiency of SN-driven outflows and that the MD of a dSph and the relationship between the stellar mass and the mean metallicity for these galaxies are direct consequences of the model. These results are in strong support of those of [17], where earlier and less precise data on metallicities of dwarf galaxies were used to address this problem.

In our approach, we consider evolution of Fe in a homogeneous system of condensed gas governed by

$$\frac{dM_g}{dt} = \left( \frac{dM_g}{dt} \right)_{\text{in}} - \psi(t) - F_{\text{out}}(t), \quad [1]$$

$$\frac{dM_{\text{Fe}}}{dt} = P_{\text{Fe}}(t) - \frac{M_{\text{Fe}}(t)}{M_g(t)} [\psi(t) + F_{\text{out}}(t)], \quad [2]$$

where  $M_g(t)$  is the mass of gas in the system at time  $t$ ,  $(dM_g/dt)_{\text{in}}$  is the infall rate of pristine gas,  $\psi(t)$  is the star formation rate (SFR),  $F_{\text{out}}(t)$  is the rate of gas outflow,  $M_{\text{Fe}}(t)$  is the mass of Fe in the gas, and  $P_{\text{Fe}}(t)$  is the net rate of Fe production by all sources in the system. We assume that the SFR is proportional to the mass of gas in the system with an astration rate constant  $\lambda_*$ ,

$$\psi(t) = \lambda_* M_g(t). \quad [3]$$

Given  $(dM_g/dt)_{\text{in}}$ ,  $F_{\text{out}}(t)$ , and  $P_{\text{Fe}}(t)$ , Eqs. 1 and 2 can be solved with the initial conditions  $M_g(0) = 0$  and  $M_{\text{Fe}}(0) = 0$ .

The MD of a system measures the numbers of stars formed in different metallicity intervals that survive until the present time. We use  $[\text{Fe}/\text{H}]$  to measure metallicity. As the mass fraction of H changes very little over the history of the universe, we take  $[\text{Fe}/\text{H}] = \log Z_{\text{Fe}}$ , where

$$Z_{\text{Fe}}(t) \equiv \frac{M_{\text{Fe}}(t)}{X_{\text{Fe}}^\odot M_g(t)}. \quad [4]$$

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Here  $X_{\text{Fe}}^{\odot}$  is the mass fraction of Fe in the sun. We assume that the initial mass function of SF does not change with time and is of the Salpeter form. Then the number of stars formed per unit mass interval per unit time is related to the SFR as

$$\frac{d^2 N}{dm dt} = \left[ \frac{\psi(t)}{M_{\odot}} \right] \frac{m^{-2.35}}{\int_{m_l}^{m_u} m^{-1.35} dm}, \quad [5]$$

where  $m$  is the stellar mass in units of  $M_{\odot}$  with  $m_l$  and  $m_u$  being the lower and upper limits, respectively. We take  $m_l = 0.1$  and  $m_u = 100$ . Assuming that  $Z_{\text{Fe}}(t)$  increases monotonically with time, we obtain the MD

$$\begin{aligned} \frac{dN}{d[\text{Fe}/\text{H}]} &= \frac{\int_{0.1}^{m_{\text{max}}(t)} (d^2 N / dm dt) dm}{d[\text{Fe}/\text{H}] / dt} \\ &= \left( \frac{\lambda_*}{\log e} \right) \frac{\int_{0.1}^{m_{\text{max}}(t)} m^{-2.35} dm}{\int_{0.1}^{100} m^{-1.35} dm} \\ &\times \left[ \frac{M_g(t)}{M_{\odot}} \right] \frac{Z_{\text{Fe}}(t)}{dZ_{\text{Fe}}/dt}, \end{aligned} \quad [6]$$

where  $m_{\text{max}}(t)$  is the maximum mass of those stars formed at time  $t$  that survive until the present time. There is little SF in dSphs at the present time (cf. [18]). We assume that SF ended at time  $t_f$  in a system. Then the total number of stars in the system at the present time is

$$\begin{aligned} N_{\text{tot}} &= \int_0^{t_f} \int_{0.1}^{m_{\text{max}}(t)} \frac{d^2 N}{dm dt} dm dt \\ &= \frac{\lambda_*}{\int_{0.1}^{100} m^{-1.35} dm} \int_0^{t_f} \frac{M_g(t)}{M_{\odot}} \int_{0.1}^{m_{\text{max}}(t)} \frac{dm dt}{m^{2.35}}. \end{aligned} \quad [7]$$

The integral involving  $m_{\text{max}}(t)$  in Eqs. 6 and 7 increases only by 6% when  $m_{\text{max}}(t)$  increases from 0.8 to 100. As stars with  $m = 0.8$  have a lifetime approximately equal to the age of the universe, we take  $m_{\text{max}}(t) = 0.8$  in both these equations to obtain the normalized MD

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{1}{\log e} \left[ \frac{M_g(t)}{\int_0^{t_f} M_g(t) dt} \right] \frac{Z_{\text{Fe}}(t)}{dZ_{\text{Fe}}/dt}. \quad [8]$$

## The Model

The key input for our model is the infall rate  $(dM_g/dt)_{\text{in}}$ , the outflow rate  $F_{\text{out}}(t)$ , and the net Fe production rate  $P_{\text{Fe}}(t)$ . The latter two rates are closely related to the occurrences of CCSNe and SNe Ia in a system. CCSNe are associated with massive stars ( $8 < m \leq 100$ ) that evolve rapidly. In contrast, SNe Ia require consideration of the evolution of binaries involving lower-mass stars with longer lifetimes. In all our previous studies (e.g., [19]), we considered that the evolution timescale for SNe Ia was  $\sim 1$  Gyr using the lifetime of stars with  $m \sim 2$ . This meant that SNe Ia would not contribute to the Fe inventory during early epochs and was in accord with the general approach used by other workers. The consequence of this assumption is that the MD for a system must have two peaks due to the assumed late onset of SNe Ia (e.g., [19]). However, of the eight dSphs studied in [14], only a single peak is observed in the MD for Fornax, Leo I, Leo II, Sextans, Draco, and Canes Venatici I, and there is only some indication for two peaks in the MD for Ursa Minor and perhaps Sculptor. From this we conclude that there must be a prompt component of SNe Ia that start in a system on much shorter timescales than  $\sim 1$  Gyr. Such a component is supported by both supernova surveys (e.g., [20]) and models that considered detailed evolution of various binary configurations (e.g.,

[21, 22]). The occurrences of “prompt” and “delayed” SNe Ia were investigated in [23], where it was argued that both populations were present at high redshift. The nature of these two classes of SNe Ia remains unclear. For simplicity, we will lump the Fe production by these sources together with that by CCSNe and ignore the time delay between the birth and death of all SN progenitors, except when the amount of gas in the system is so low that SNe Ia would dominate. A detailed treatment of SNe Ia that takes into account their evolution timescales will be discussed in a subsequent paper.

Under our assumption, the total rate of CCSNe and SNe Ia, and hence  $P_{\text{Fe}}(t)$ , are proportional to the SFR. We further assume that the rate of outflows driven by these SNe is also proportional to the SFR. Specifically, we take

$$F_{\text{out}}(t) = \eta \lambda_* M_g(t), \quad [9]$$

$$P_{\text{Fe}}(t) = \lambda_{\text{Fe}} X_{\text{Fe}}^{\odot} M_g(t), \quad [10]$$

where  $\eta$  is a dimensionless constant that measures the efficiency of the SN-driven outflows and  $\lambda_{\text{Fe}}$  is a rate constant that is proportional to  $\lambda_*$  and the effective Fe yield of SNe. We take the infall rate to be

$$\left( \frac{dM_g}{dt} \right)_{\text{in}} = \lambda_{\text{in}} M_0 \frac{(\lambda_{\text{in}} t)^{\alpha}}{\Gamma(\alpha + 1)} \exp(-\lambda_{\text{in}} t), \quad [11]$$

where  $\lambda_{\text{in}}$  is a rate constant,  $M_0$  is the total mass of gas infall over  $0 \leq t < \infty$ , and  $\Gamma(\alpha + 1)$  with  $\alpha > -1$  is the Gamma function of argument  $\alpha + 1$ . The above modified exponential form was specifically chosen to explore the role of the time dependence of the infall rate in chemical evolution (cf. [7]). The infall rate peaks at  $t = 0$  for  $-1 < \alpha \leq 0$ , and the peak time increases to  $\alpha/\lambda_{\text{in}}$  for  $\alpha > 0$ . The form with  $\alpha > 0$  allows a slow start of significant gas accumulation in the system.

With the above assumptions, Eqs. 1 and 2 become

$$\frac{dM_g}{dt} = \lambda_{\text{in}} M_0 \frac{(\lambda_{\text{in}} t)^{\alpha}}{\Gamma(\alpha + 1)} \exp(-\lambda_{\text{in}} t) - \lambda M_g(t), \quad [12]$$

$$\frac{dM_{\text{Fe}}}{dt} = \lambda_{\text{Fe}} X_{\text{Fe}}^{\odot} M_g(t) - \lambda M_{\text{Fe}}(t), \quad [13]$$

where  $\lambda \equiv (1 + \eta)\lambda_*$ . Note that for  $\lambda_{\text{in}} \ll \lambda$ , the solutions to the above equations approach a secular state for which  $M_g(t) \approx (dM_g/dt)_{\text{in}}/\lambda$  and  $Z_{\text{Fe}} \approx \lambda_{\text{Fe}}/\lambda$ . This is analogous to the quasi-steady state for the metallicity of the interstellar medium first proposed in [24]. For simplicity, we assume that  $\lambda_{\text{in}}$  and  $\lambda$  are so large that  $t_f = \infty$  can be used effectively in Eq. 8 for the MD. As  $M_g(0) = M_g(\infty) = 0$ , integrating Eq. 12 over  $t$  gives  $\lambda \int_0^{\infty} M_g(t) dt = M_0$ . So the total gas mass used in SF is  $\lambda_* \int_0^{\infty} M_g(t) dt = M_0/(1 + \eta)$  and the remainder of the gas infall is blown out as outflows. We assume that all outflows are lost from the system into the broader intergalactic medium (IGM), thus enriching the latter in metals. This approach gives a natural cutoff to the chemical evolution of the system when the total mass of gas lost to SF and outflows is equal to the total mass of gas infall (partial recycling of outflows would increase the degree of chemical enrichment but is ignored here for simplicity). At the present time, the total mass of stars in the system can be estimated as

$$M_* = \frac{\int_{0.1}^{0.8} m^{-1.35} dm}{\int_{0.1}^{100} m^{-1.35} dm} \frac{M_0}{1 + \eta} = 9.65 \times 10^{-2} \left( \frac{M_h}{1 + \eta} \right), \quad [14]$$

where we have used  $M_0 = (\Omega_b/\Omega_m)M_h = 0.17M_h$ . Here  $M_h$  is the total mass inside the dark matter halo hosting the system, and  $\Omega_b$  and  $\Omega_m$  are the fractional contributions to the critical density of the universe from baryonic and all matter, respectively.

## Exploration of Results from the Model

The parameters governing the solutions to Eqs. 12 and 13 are  $\alpha$ ,  $\lambda_{\text{in}}$ ,  $\lambda$ , and  $\lambda_{\text{Fe}}$ . We illustrate the dependences of the MD on these parameters in the following subsections.

**Dependence of the MD on  $\lambda_{\text{in}}/\lambda$  with  $\alpha = 0$ .** For  $\alpha = 0$ , the solutions to Eqs. 12 and 13 are

$$M_g(t) = \frac{\lambda_{\text{in}}}{\lambda_{\text{in}} - \lambda} M_0 [\exp(-\lambda t) - \exp(-\lambda_{\text{in}} t)], \quad [15]$$

$$M_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}} \lambda_{\text{in}}}{(\lambda_{\text{in}} - \lambda)^2} X_{\text{Fe}}^{\odot} M_0 \{ \exp(-\lambda_{\text{in}} t) + [(\lambda_{\text{in}} - \lambda)t - 1] \exp(-\lambda t) \}, \quad [16]$$

from which we obtain

$$Z_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}}}{\lambda_{\text{in}} - \lambda} \left[ \frac{(\lambda_{\text{in}} - \lambda)t}{1 - \exp[-(\lambda_{\text{in}} - \lambda)t]} - 1 \right] \quad [17]$$

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{1}{\log e} \frac{\lambda_{\text{in}} \lambda}{(\lambda_{\text{in}} - \lambda)^2} \frac{\{1 - \exp[-(\lambda_{\text{in}} - \lambda)t]\}^2}{\exp(\lambda t)} \times \frac{(\lambda_{\text{in}} - \lambda)t - 1 + \exp[-(\lambda_{\text{in}} - \lambda)t]}{1 - [1 + (\lambda_{\text{in}} - \lambda)t] \exp[-(\lambda_{\text{in}} - \lambda)t]} \quad [18]$$

For  $\lambda_{\text{in}}/\lambda = 1$ , the above two equations reduce to

$$Z_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}} t}{2}, \quad [19]$$

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{(\lambda t)^2}{\log e} \exp(-\lambda t). \quad [20]$$

In the limit  $\lambda_{\text{in}} = \infty$ , which is equivalent to setting  $M_g(0) = M_0$  and  $(dM_g/dt)_{\text{in}} = 0$  for  $t > 0$ , the results are

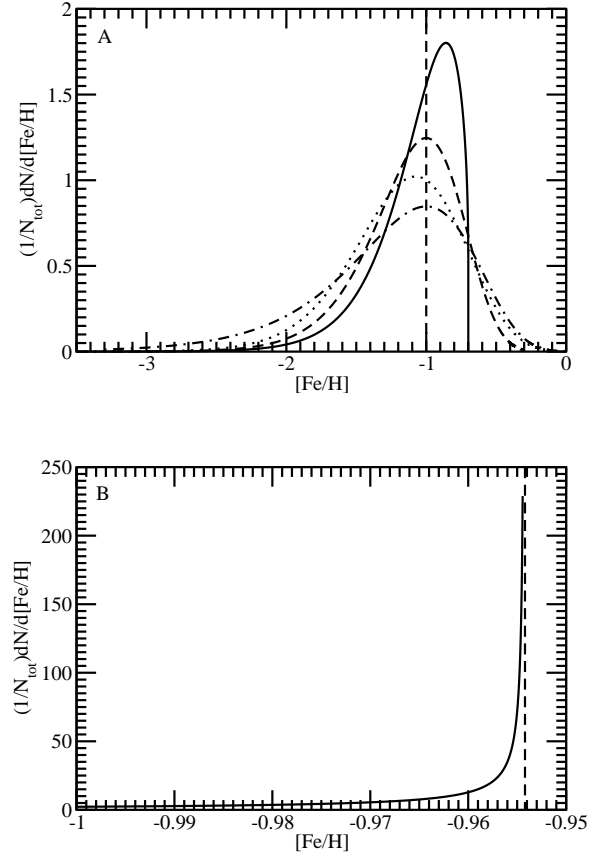
$$Z_{\text{Fe}}(t) = \lambda_{\text{Fe}} t, \quad [21]$$

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{\lambda t}{\log e} \exp(-\lambda t). \quad [22]$$

In general, Eqs. 17 and 18 give the MD as a function of  $[\text{Fe}/\text{H}]$  in parametric form. This MD only depends on  $\lambda_{\text{in}}/\lambda$  and  $\lambda_{\text{Fe}}/\lambda$ , but not on the absolute values of these rates (this is true so long as  $\lambda t_f \gg 1$  and  $\lambda_{\text{in}} t_f \gg 1$ , see Eqs. 8 and 15). For a fixed  $\lambda_{\text{in}}/\lambda$ , changing  $\lambda_{\text{Fe}}/\lambda$  only translates the MD along the  $[\text{Fe}/\text{H}]$ -axis. This can be most easily seen in the special cases of  $\lambda_{\text{in}}/\lambda = 1$  and  $\lambda_{\text{in}} = \infty$ , where  $N_{\text{tot}}^{-1} dN/d[\text{Fe}/\text{H}]$  is a simple function of  $\lambda t$  and  $[\text{Fe}/\text{H}]$  differs from  $\log(\lambda t)$  only by a shift of  $\log(\lambda_{\text{Fe}}/2\lambda)$  and  $\log(\lambda_{\text{Fe}}/\lambda)$ , respectively. The shape of the MD is determined by  $\lambda_{\text{in}}/\lambda$ . This can be seen from Fig. 1A, which uses  $\lambda_{\text{Fe}}/\lambda = 0.1$  and shows that as  $\lambda_{\text{in}}/\lambda$  increases from  $1/2$  to  $\infty$ , the position of the peak of the MD changes slightly [but staying close to  $[\text{Fe}/\text{H}] = \log(\lambda_{\text{Fe}}/\lambda) = -1$ , the exact peak position for  $\lambda_{\text{in}}/\lambda = 1$  and  $\lambda_{\text{in}} = \infty$ ] and the shape of the MD becomes broader. Note the sharp cutoff of the MD to the right of the peak for  $\lambda_{\text{in}}/\lambda = 1/2$  with no stars formed above  $[\text{Fe}/\text{H}] = \log[\lambda_{\text{Fe}}/(\lambda - \lambda_{\text{in}})] = -0.7$ .

The case of  $\lambda_{\text{in}}/\lambda < 1/2$  requires separate discussion. For illustration, we again take  $\lambda_{\text{Fe}}/\lambda = 0.1$  and show the MD for  $\lambda_{\text{in}}/\lambda = 0.1$  in Fig. 1B. This MD has an extremely sharp peak with 90% of the stars having  $-1.154 \leq [\text{Fe}/\text{H}] < -0.954$ . This is simply a case close to secular equilibrium (see discussion below Eqs. 12 and 13; cf. [24]). Piling up of stars in an extremely narrow metallicity range is typically not observed for dwarf galaxies but strongly resembles what is observed for most globular clusters. In general, for  $\lambda_{\text{in}}/\lambda$  significantly below  $1/2$ , stars are concentrated immediately below  $[\text{Fe}/\text{H}] = \log[\lambda_{\text{Fe}}/(\lambda - \lambda_{\text{in}})]$ . For comparison, the solid curve in Fig. 1A shows that the MD for  $\lambda_{\text{in}}/\lambda = 1/2$  first rises to a peak and

then sharply drops to zero as  $[\text{Fe}/\text{H}] \rightarrow \log[\lambda_{\text{Fe}}/(\lambda - \lambda_{\text{in}})]$ . This general behavior also applies to  $\lambda_{\text{in}}/\lambda > 1/2$  but with a more extended tail at high metallicities for a larger  $\lambda_{\text{in}}/\lambda$  (see Fig. 1A).



**Fig. 1.** Example MDs for  $\lambda_{\text{Fe}}/\lambda = 0.1$  and various values of  $\lambda_{\text{in}}/\lambda$  with  $\alpha = 0$ . Note that in general the shape of the MD is determined by  $\lambda_{\text{in}}/\lambda$  and changing  $\lambda_{\text{Fe}}/\lambda$  only translates the MD along the  $[\text{Fe}/\text{H}]$ -axis. (A) The solid, dashed, dotted, and dot-dashed curves are the MDs for  $\lambda_{\text{in}}/\lambda = 1/2, 1, 2$ , and  $\infty$ , respectively. The vertical dashed line indicates the peak at  $[\text{Fe}/\text{H}] = \log(\lambda_{\text{Fe}}/\lambda) = -1$  for the dashed and dot-dashed curves. (B) The solid curve is the MD for  $\lambda_{\text{in}}/\lambda = 0.1$  and the dashed line indicates the limiting value of  $[\text{Fe}/\text{H}] = \log[\lambda_{\text{Fe}}/(\lambda - \lambda_{\text{in}})] = -0.954$ . Concentration of stars in an extremely narrow range of  $[\text{Fe}/\text{H}]$  as shown by the solid curve is typically not observed for dSphs but strongly resembles what is observed for most globular clusters.

**Dependence of the MD on  $\alpha$  with  $\lambda_{\text{in}} = \lambda$ .** Based on the above discussion and the example MDs shown in Fig. 1, it seems reasonable to choose  $\lambda_{\text{in}} = \lambda$  and explore possible MDs for different values of  $\alpha$  and  $\lambda_{\text{Fe}}/\lambda$ . For  $\lambda_{\text{in}} = \lambda$  and  $\alpha > -1$ , the solutions to Eqs. 12 and 13 are

$$M_g(t) = M_0 \frac{(\lambda t)^{\alpha+1}}{\Gamma(\alpha+2)} \exp(-\lambda t), \quad [23]$$

$$M_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}}}{\lambda} X_{\text{Fe}}^{\odot} M_0 \frac{(\lambda t)^{\alpha+2}}{\Gamma(\alpha+3)} \exp(-\lambda t), \quad [24]$$

which give

$$Z_{\text{Fe}}(t) = \frac{\lambda_{\text{Fe}} t}{\alpha + 2}, \quad [25]$$

$$\frac{1}{N_{\text{tot}}} \frac{dN}{d[\text{Fe}/\text{H}]} = \frac{1}{\log e} \frac{(\lambda t)^{\alpha+2}}{\Gamma(\alpha+2)} \exp(-\lambda t). \quad [26]$$

The above MD again has a peak at  $[\text{Fe}/\text{H}] = \log(\lambda_{\text{Fe}}/\lambda)$  for all  $\alpha > -1$ . Changing  $\lambda_{\text{Fe}}/\lambda$  only translates the MD, shifting the peak in particular, along the  $[\text{Fe}/\text{H}]$ -axis. The shape of the MD is determined by  $\alpha$ . For a specific  $\lambda_{\text{Fe}}/\lambda$ , the MD becomes narrower while peaking at the same  $[\text{Fe}/\text{H}]$  when  $\alpha$  increases above  $-1$ . A positive  $\alpha$  reflects slower infall at the start of the system, which suppresses SF at the early stages and reduces the extent of the low-metallicity tail of the MD. For  $\alpha < 0$ , the initial infall rate is enhanced and consequently, more stars are formed at lower metallicities.

Note that if we take the limit  $\alpha = -1$ , the MD given by Eqs. 25 and 26 coincides with that for  $\lambda_{\text{in}} = \infty$  and  $\alpha = 0$  (see Eqs. 21 and 22). Thus, for  $\lambda_{\text{Fe}}/\lambda = 0.1$ , as  $\alpha$  increases from  $-1$  to  $0$ , the MD for  $\lambda_{\text{in}} = \lambda$  changes from the dot-dashed to the dashed curve shown in Fig. 1A while peaking at the same  $[\text{Fe}/\text{H}] = -1$ . More example MDs for  $\lambda_{\text{in}} = \lambda$  and  $\alpha \geq -1$  are shown in Fig. 2 where we compare them with observations of dSphs.

### Comparison with Observations

We now explore the implications of our model for observations of dSphs. We first discuss the MDs using the high-quality data set of [14] and then study the relationship between the mean metallicity  $\langle[\text{Fe}/\text{H}]\rangle$  and the stellar mass  $M_*$  of dSphs.

**MDs for dSphs.** Important medium-resolution data on the MDs for dSphs were provided in [14] and are summarized in Fig. 2. Careful inspection of these data shows that there is only some indication for two peaks in the MD for Ursa Minor and perhaps Sculptor. An MD with two peaks would be typical if the turn-on of SNe Ia were sudden and with a significant delay relative to CCSNe. It was the lack of such MDs from observations that led us to pursue a model where the net Fe production rate is without discontinuities.

We focus on models with  $\lambda_{\text{in}} = \lambda$  and different values of  $\alpha$ . In this case, a model MD is specified by  $\lambda_{\text{Fe}}/\lambda$  and  $\alpha$ , which determine its peak position  $[\text{Fe}/\text{H}] = \log(\lambda_{\text{Fe}}/\lambda)$  and its shape, respectively. Positive values of  $\alpha$  corresponding to slower initial infall give rise to narrower MDs while negative values corresponding to more rapid initial infall result in more extended MDs. As the MD is normalized, its peak height can be used to estimate  $\alpha$  effectively. Using the position and the height of the peak for the observed MD as guides, we fit a model MD for each of the eight dSphs reported in [14]. The observed and fitted MDs are shown as histograms and curves, respectively, in Fig. 2. The adopted values of  $\lambda_{\text{Fe}}/\lambda$  and  $\alpha$  are indicated for each galaxy. These values were not obtained from the best fits, but were simply picked to illustrate the overall adequacy of our model. Very good fits are obtained for Fornax and Leo I, which are the most massive of the eight dSphs. The fits for the least massive four, Draco, Sextans, Ursa Minor, and Canes Venatici I, are rather good, although the model MDs appear to underestimate their stellar populations at the highest metallicities. The only exceptions are Leo II and Sculptor with intermediate  $M_*$ . Using the same  $\lambda_{\text{Fe}}/\lambda$ , we obtained a better fit to the data for Leo II with  $\lambda_{\text{in}}/\lambda = 0.8$  and  $\alpha = 0$  (dashed curve in Fig. 2C) than with  $\lambda_{\text{in}}/\lambda = 1$  and  $\alpha = 1$  (solid curve). However, neither type of MD can provide a good fit to the data for Sculptor, which may represent an MD with two peaks. There is perhaps a more clear indica-

tion for such an MD for Ursa Minor. We note that if infall took a more complicated form than Eq. 11, then the shape of the MD would change accordingly. It is possible that the broadened peak in the MD for Sculptor might be explained by an increase in the infall rate after the assumed smooth rate in Eq. 11 peaked. However, in no case can a significant part of the enriched outflows be returned to the infalling matter as this would produce a population of stars with high  $[\text{Fe}/\text{H}]$  values, which is not observed.

In a previous study [14] with alternative modelling, detailed multi-parameter fitting was used. As can be seen from Fig. 2, a good to excellent fit of the model to the data can be obtained in our approach. The model thus appears to give a good description of the MDs of dSphs and is easily understood in terms of physical processes.

**Relationship between  $\langle[\text{Fe}/\text{H}]\rangle$  and  $M_*$  for dSphs.** The observed MDs of dSphs require that  $\lambda_{\text{Fe}}/\lambda \sim 0.01\text{--}0.1$  (see Fig. 2). As  $\lambda_{\text{Fe}}/\lambda \sim (1 + \eta)^{-1}$  (see derivation of  $\lambda_{\text{Fe}}$  in SI Text), this requires that  $\eta \sim 10\text{--}100$ . Consequently, only a fraction  $(1 + \eta)^{-1} \sim 1\text{--}10\%$  of the gas falling into the dark matter halo hosting a dSph is used in SF and the rest is blown out of the halo by SN-driven outflows. The MD of a dSph peaks at essentially its mean metallicity  $\langle[\text{Fe}/\text{H}]\rangle \approx \log(\lambda_{\text{Fe}}/\lambda) \sim -\log \eta$ . The lower  $\langle[\text{Fe}/\text{H}]\rangle$  a dSph has, the lower fraction of the infalling gas is stored in its stars. There is thus a direct relationship between  $\langle[\text{Fe}/\text{H}]\rangle$  and the stellar mass  $M_*$  for dSphs. Using the data on  $\langle[\text{Fe}/\text{H}]\rangle$  in [14] and those on  $M_*$  in [25, 26], we show this relationship in Fig. 2. The dot-dashed line in Fig. 2 has a slope of 2.5 and passes through the point defined by the average values of the data. It is nearly the same as the least-square fit (solid line) and is in excellent agreement with the result in [25] (see SI Text).

As discussed in [17], the slope of 2.5 for the relationship between  $\log(M_*/M_\odot)$  and  $\langle[\text{Fe}/\text{H}]\rangle$  has a simple physical explanation. In general, this slope follows from the relationship between the radius  $r_h$  and the total mass  $M_h$  of the dark matter halo hosting a dSph in addition to the dependences of  $M_*$  and  $\langle[\text{Fe}/\text{H}]\rangle$  on  $\eta$  discussed above. The outflow efficiency  $\eta$  is inversely proportional to the depth of the gravitational potential well of the dark matter halo,

$$\eta \propto r_h/M_h \propto M_h^{\beta-1}, \quad [27]$$

where we have assumed  $r_h \propto M_h^\beta$  in the second step. This gives

$$\langle[\text{Fe}/\text{H}]\rangle \sim -\log \eta \sim (1 - \beta) \log M_h + \text{const.} \quad [28]$$

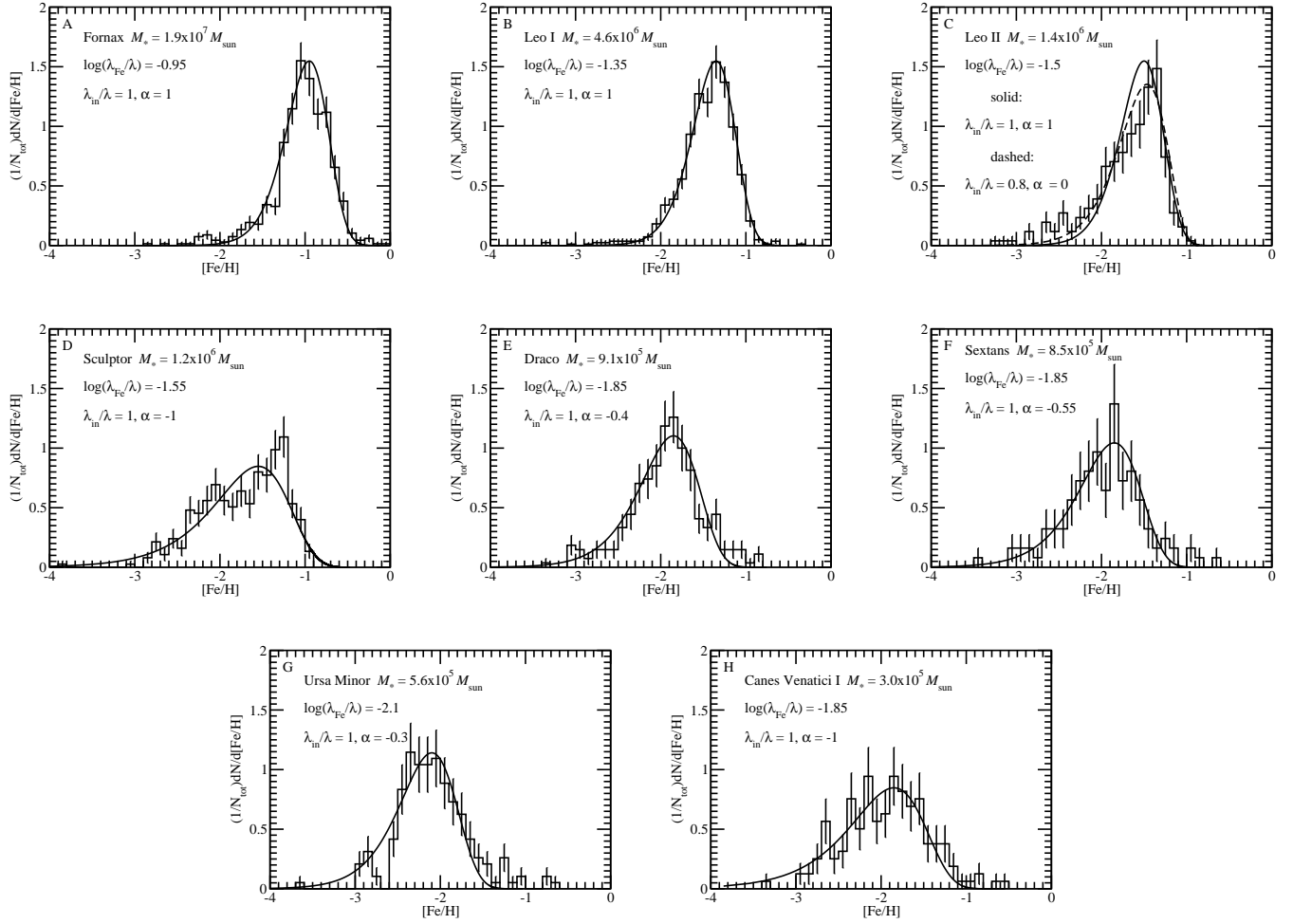
In addition, Eq. 14 gives

$$M_* \propto M_h/\eta \propto M_h^{2-\beta}. \quad [29]$$

Thus, a slope of 2.5 for the relationship between  $\log(M_*/M_\odot)$  and  $\langle[\text{Fe}/\text{H}]\rangle$  requires  $(2 - \beta)/(1 - \beta) = 2.5$ , or  $\beta = 1/3$ , which is in agreement with the framework of hierarchical structure formation (e.g., Eq. 24 in [27]).

### Conclusions

We have shown that the MDs of dSphs can be reasonably well described quantitatively by a phenomenological model based on general considerations of gas infall into a dark matter halo, astration in the condensed gas, and SN-driven gas outflows. The peak position of the MD is governed by the ratio of the rate constants for net Fe production and net gas loss to astration and outflows,  $\lambda_{\text{Fe}}/\lambda = \lambda_{\text{Fe}}/[(1 + \eta)\lambda_*]$ . The observations require high efficiency  $\eta \sim 10\text{--}360$  (see Table S1) for SN-driven



**Fig. 2.** Comparison of model MDs with observations of dSphs. The data are taken from [14] and shown as histograms with error bars. The model MDs assume the indicated parameters and are shown as curves. The dashed curve in (C) provides a better fit to the data than the solid curve. Values of  $M_*$  are taken from [25] for (A)–(G) and from [26] for (H). Note that if the baryonic matter is not always blown out of the dark matter halo but returns to the gas mass in a dSph after some time, then the curves will have a leading-edge tail going to higher  $[\text{Fe}/\text{H}]$ .

outflows and hence, massive gas loss from dark matter halos associated with dSphs. The model also directly relates the stellar mass  $M_*$  remaining in a dSph to its mean metallicity  $\langle[\text{Fe}/\text{H}]\rangle$  through the efficiency of outflows governed by the mass  $M_h$  and the radius  $r_h$  of the dark matter halo. The observed relationship between  $\log(M_*/M_\odot)$  and  $\langle[\text{Fe}/\text{H}]\rangle$  has a slope of 2.5 over the wide range of  $4.8 \times 10^3 \leq M_*/M_\odot \leq 4.6 \times 10^8$  for dwarf galaxies (see SI Text). This indicates  $r_h \propto M_h^{1/3}$  in agreement with the framework of hierarchical structure formation. Our results confirm the previous studies of [17] and [25], and are in support of the early work of [13] on the general model of dwarf galaxy formation.

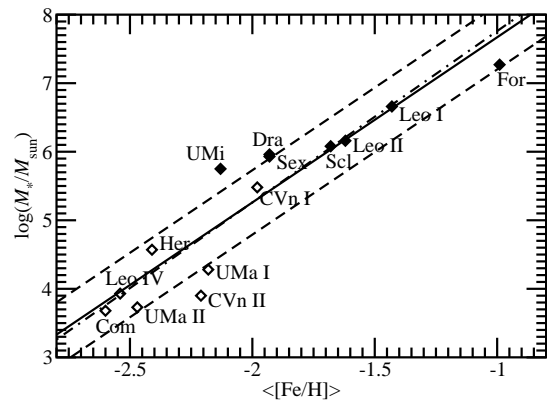
Our model also demonstrates that for slow infall rates  $\lambda_{\text{in}} \ll \lambda$ , the resulting MD must be extremely sharply peaked (essentially concentrated at a single value of  $[\text{Fe}/\text{H}]$ ). Such MDs apply to all globular clusters with the exception of the most massive ones (e.g., [28]). This suggests that globular clusters are the result of the general process of astration governed by very slow infall rates compared to net gas loss rates. These slow infall rates further suggest that globular clusters might be weak feeders during the inhomogeneous evolution inside a large system and that they are not responsible for sig-

nificant depletion of the baryonic supply of that system. As such, the dark matter halo would be associated with the entire system, but not be specifically tied to the globular clusters. This is in contrast to dSphs, which result from processing all the baryonic supply in their dark matter halos and are naturally associated with these halos. Nonetheless, from the point of view of the analysis presented here, we consider globular clusters to be part of a family of early-formed dwarf galaxies but with low effective infall rates compared to outflow rates. With regard to the diverse morphological types of dwarf galaxies, we consider dSphs to represent isolated evolution without dynamic effects from mergers or tidal interactions with nearby systems. Other morphological types may result from such effects. Insofar as all dwarf galaxies are related by the same general process as presented here, the observed ongoing astration in some dwarf irregular galaxies (e.g., [18]) poses a problem. These systems must be experiencing secondary processes of gas accretion due to local infall or mergers. Their MDs should have a second peak due to the late infall.

The analysis presented here assumes that there is not a discontinuous onset of SNe Ia contributing Fe. Otherwise, the general outcome would be MDs with two peaks, which are at

most only rarely observed. It is not appropriate to extend this analysis to other metals than Fe without a more realistic treatment of the relative contributions of CCSNe and SNe Ia that each produce very different yields of the other metals. Such a treatment will be carried out in a subsequent paper that takes into account the detailed distribution of the delay between the birth and death of the progenitors for SNe Ia. We note here that as the rate of CCSNe decreases with decreasing gas mass, the contributions to Fe from SNe Ia will become larger or dominant. This is because previously-formed stars of low to intermediate masses in binaries will continue to evolve and produce SNe Ia even after the SFR decreases. Crudely speaking, the net Fe production rate appropriate for later times must be changed from Eq. 10 to  $P_{\text{Fe}}(t) \sim X_{\text{Fe}}^{\odot} [\lambda_{\text{Fe}}^{\text{CC}} M_g(t) + \lambda_{\text{Fe}}^{\text{Ia}} M_g(t - \Delta)]$ , where  $\lambda_{\text{Fe}}^{\text{CC}}$  and  $\lambda_{\text{Fe}}^{\text{Ia}}$  are the rate constants for CCSNe and SNe Ia, respectively, and  $\Delta$  is a typical delay between the birth and death of the progenitors for SNe Ia. As SNe Ia do not produce e.g., O, Mg, and Si, this will result in lower values of [E/Fe] for these elements at higher values of [Fe/H], which is in agreement with the general trend observed in dSphs (e.g., [15]).

IGM must have been enriched by the net outflows from all dSphs. If the average IGM has  $[\text{Fe}/\text{H}] \sim -3$  and the outflows from dSphs have  $[\text{Fe}/\text{H}] \sim -1.5$  on average, this would imply that  $\sim 3\%$  of all baryonic matter was processed in dSphs.



**Fig. 3.** The relationship between  $\log(M_*/M_\odot)$  and  $\langle[\text{Fe}/\text{H}]\rangle$  for dSphs. Values of  $M_*$  for filled and open diamonds are taken from [25] and [26], respectively. All values of  $\langle[\text{Fe}/\text{H}]\rangle$  used here are taken from [14]. The solid line is the least-square fit to the data and the dashed lines indicate the  $1\sigma$  error in  $\log(M_*/M_\odot)$ . The dot-dashed line has a slope of 2.5 and is nearly the same as the solid line.

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## SI Text

**Relationship between  $\langle[\text{Fe}/\text{H}]\rangle$  and  $M_*$  for dSphs.** The data on  $\langle[\text{Fe}/\text{H}]\rangle$  and  $M_*$  for the eight dSphs shown in Fig. 2 plus another six from literature are given in Table S1 and shown in Fig. 3. All values of  $\langle[\text{Fe}/\text{H}]\rangle$  are taken from [1]. The values of  $M_*$  for Fornax, Leo I, Leo II, Sculptor, Draco, Sextans, and Ursa Minor (filled diamonds in Fig. 3) are taken from [2] while those for Canes Venatici I, Hercules, Ursa Major I, Leo IV, Canes Venatici II, Ursa Major II, and Coma Berenices (open diamonds) are taken from [3]. (An important input to infer the  $M_*$  of a dwarf galaxy is the present mass distribution of its stellar populations. Two different mass distributions were used in [3], which gave values of  $M_*$  differing by a factor of  $\approx 2$ . For consistency, we have adopted the results derived in [3] from the mass distribution that is similar to the one used in [2].) These values of  $M_*$  are inferred from observations and have some considerable uncertainties. For example, Draco is common to the data sets of [2] and [3] and is inferred to have  $M_* = 9.1 \times 10^5 M_\odot$  and  $3.2 \times 10^5 M_\odot$ , respectively. In view of this, we treat  $\log(M_*/M_\odot)$  as a function of the better determined  $\langle[\text{Fe}/\text{H}]\rangle$  and obtain a least-square linear fit

$$\log(M_*/M_\odot) = 10.08 \pm 0.59 + (2.41 \pm 0.29)\langle[\text{Fe}/\text{H}]\rangle. \quad [1]$$

The solid line in Fig. 3 corresponds to the central values of the above fit. We estimate that the  $1\sigma$  error in the inferred values of  $\log(M_*/M_\odot)$  is 0.47 dex. This error is indicated by the two dashed lines in Fig. 3.

The relationship in Eq. S1 is essentially the same as that found in [2], which used a set of more massive dwarf galaxies ( $4.0 \times 10^5 \leq M_*/M_\odot \leq 4.6 \times 10^8$ , including both irregular and spheroidal galaxies) and earlier and less precise data on  $\langle[\text{Fe}/\text{H}]\rangle$ . That work also used a different parameter  $\log Z \equiv \langle[\text{Fe}/\text{H}]\rangle + \log Z_\odot$ , where  $Z_\odot = 0.019$  is the total mass fraction of metals in the sun [4]. In the representation used here, [2] obtained

$$\log(M_*/M_\odot) = 10.37 + 2.5\langle[\text{Fe}/\text{H}]\rangle. \quad [2]$$

Our result in Eq. S1 for a different mass range ( $4.8 \times 10^3 \leq M_*/M_\odot \leq 1.9 \times 10^7$ ) using only dSphs is in excellent agreement with Eq. S2. The dot-dashed line in Fig. 3 has a slope of 2.5 and passes through the point defined by the average values of the data. It corresponds to

$$\log(M_*/M_\odot) = 10.26 + 2.5\langle[\text{Fe}/\text{H}]\rangle, \quad [3]$$

and is nearly the same as the solid line. The mass ranges covered by Eqs. S2 and S3 overlap and both include the seven dSphs shown as filled diamonds in Fig. 3. Together these two results show that  $\log(M_*/M_\odot)$  increases with  $\langle[\text{Fe}/\text{H}]\rangle$  with a slope of 2.5 over the wide range of  $4.8 \times 10^3 \leq M_*/M_\odot \leq 4.6 \times 10^8$  for dwarf galaxies.

**Parameters of the Model.** We can estimate the parameters of our model based on properties of the dark matter halo hosting

a dSph and characteristics of CCSNe and SNe Ia. The rate of CCSNe is

$$R_{\text{CC}}(t) = \int_8^{100} \frac{d^2 N}{dm dt} dm = 7.42 \times 10^{-3} \lambda_* \left[ \frac{M_g(t)}{M_\odot} \right]. \quad [4]$$

For simplicity, we assume that the rate of SNe Ia is  $R_{\text{Ia}}(t) = k R_{\text{CC}}(t)$ , where  $k$  is a constant. The average Fe yield of each CCSN is

$$\langle Y_{\text{Fe}}^{\text{CC}} \rangle = \frac{\int_8^{100} Y_{\text{Fe}}^{\text{CC}}(m) m^{-2.35} dm}{\int_8^{100} m^{-2.35} dm} = 4.58 \times 10^{-2} M_\odot, \quad [5]$$

where  $Y_{\text{Fe}}^{\text{CC}}(m)$  is the Fe yield of a CCSN from a progenitor of mass  $m$  and we have taken  $Y_{\text{Fe}}^{\text{CC}}(m) = 3 \times 10^{-3} M_\odot$  for  $8 < m \leq 11$  [5] and  $7 \times 10^{-2} M_\odot$  for  $11 < m \leq 100$  (e.g., [6]) in the second step. We take the average Fe yield of each SNe Ia to be  $\langle Y_{\text{Fe}}^{\text{Ia}} \rangle = 0.7 M_\odot$  (e.g., [7]). The net Fe production rate is

$$P_{\text{Fe}}(t) = \lambda_{\text{Fe}} X_{\text{Fe}}^\odot M_g(t) = \langle Y_{\text{Fe}}^{\text{CC}} \rangle R_{\text{CC}}(t) + \langle Y_{\text{Fe}}^{\text{Ia}} \rangle R_{\text{Ia}}(t). \quad [6]$$

Using the above results and  $X_{\text{Fe}}^\odot = 1.3 \times 10^{-3}$  [4], we obtain

$$\lambda_{\text{Fe}} = 0.261(1 + 15.3k)\lambda_*. \quad [7]$$

As both CCSNe and SNe Ia produce Fe, the assumption that  $R_{\text{Ia}}(t)/R_{\text{CC}}(t)$  is a constant  $k$  does not critically affect the model presented here for Fe in consideration of the approximations made. However, other elements such as O, Mg, and Si are produced by CCSNe but not by SNe Ia. For these elements the detailed distribution of the delay between the birth and death of the progenitors for SNe Ia must be taken into account. This will be dealt with in a subsequent paper that treats a number of elements including Mg and Fe with detailed considerations of SNe Ia.

The rate of SN-driven outflows is  $F_{\text{out}}(t) = \eta \lambda_* M_g(t)$  and can be estimated by

$$\frac{GM_h}{r_h} F_{\text{out}}(t) = \langle E_{\text{kin}}^{\text{CC}} \rangle R_{\text{CC}}(t) + \langle E_{\text{kin}}^{\text{Ia}} \rangle R_{\text{Ia}}(t), \quad [8]$$

**Reserved for Publication Footnotes**



where  $G$  is the gravitational constant, and  $\langle E_{\text{kin}}^{\text{CC}} \rangle$  and  $\langle E_{\text{kin}}^{\text{Ia}} \rangle$  are the average kinetic energy imparted to the surrounding gas by each CCSN and SN Ia, respectively. Specifically,

$$\langle E_{\text{kin}}^{\text{CC}} \rangle = \frac{\int_8^{100} E_{\text{kin}}^{\text{CC}}(m) m^{-2.35} dm}{\int_8^{100} m^{-2.35} dm} = 6.75 \times 10^{49} \text{ erg}, \quad [9]$$

where  $E_{\text{kin}}^{\text{CC}}(m)$  is the kinetic energy imparted to the surrounding gas by a CCSN from a progenitor of mass  $m$ , and we have taken  $E_{\text{kin}}^{\text{CC}}(m) = 10^{49}$  erg for  $8 < m \leq 11$  and  $10^{50}$  erg for  $11 < m \leq 100$  in the second step. This assumes that 10% of the explosion energy of each SN (e.g., [6, 8]) is used to drive the bulk motion of the surrounding gas. We take  $\langle E_{\text{kin}}^{\text{Ia}} \rangle = 10^{50}$  erg. Using the above results, we obtain

$$\eta = 58.5(1 + 1.48k) \left( \frac{r_h}{\text{kpc}} \right) \left( \frac{10^8 M_\odot}{M_h} \right). \quad [10]$$

According to the theory of hierarchical structure formation in cold dark matter cosmology, a halo with a total mass  $M_h$  collapsing at redshift  $z > 1$  has a radius

$$r_h = 1.54 \left( \frac{M_h}{10^8 M_\odot} \right)^{1/3} \left( \frac{10}{1+z} \right) \text{ kpc}, \quad [11]$$

which can be obtained from Eq. 24 of [9] for the current cosmological parameters ( $\Omega_m = 0.27$  and a Hubble constant of

70 km s<sup>-1</sup> Mpc<sup>-1</sup>). As was recognized in [10], this purely theoretical result is in excellent agreement with what is required (i.e.,  $r_h \propto M_h^{1/3}$ ) to account for the slope of 2.5 for the observed relationship between  $\log(M_*/M_\odot)$  and  $\langle [\text{Fe}/\text{H}] \rangle$  for dSphs. We note that there is no a priori basis for choosing an explicit time (i.e.,  $z$ ) for the formation of dSphs. It is most plausible that they formed early in cold dark matter cosmology. Taking  $1+z = 10$  and substituting  $r_h$  from Eq. S11 into Eq. S10, we obtain

$$\eta = 90.1(1 + 1.48k) \left( \frac{10^8 M_\odot}{M_h} \right)^{2/3}. \quad [12]$$

In our model,  $\langle [\text{Fe}/\text{H}] \rangle \approx \log\{\lambda_{\text{Fe}}/[(1+\eta)\lambda_*]\} \approx \log[0.261(1 + 15.3k)/\eta]$  (see Eq. S7) and  $M_* \approx 9.65 \times 10^{-2} M_h/\eta$  (see Eq. 14 in the main text). As  $\eta$  is determined by  $M_h$  and  $k$  through Eq. S12,  $\langle [\text{Fe}/\text{H}] \rangle$  and  $M_*$  are functions of  $M_h$  and  $k$ . We estimate  $M_h$  and  $k$  from the data on  $\langle [\text{Fe}/\text{H}] \rangle$  and  $M_*$  for each of the dSphs shown in Fig. 3 and give the results in Table S1. As expected, these dSphs have very strong SN-driven outflows with  $\eta \sim 10$ –360. The estimated values of  $M_h$  range from  $1.8 \times 10^7 M_\odot$  to  $2.8 \times 10^9 M_\odot$  and are  $\sim 3$ –70 times larger than the total mass within the half-light radius  $M_{1/2}$  inferred from observations [11]. The estimated values of  $k$  range from 0.02 to 0.48 and are typically  $\sim 0.1$ . In particular, the relationship between  $\log(M_*/M_\odot)$  and  $\langle [\text{Fe}/\text{H}] \rangle$  in Eq. S3 corresponds to  $k = 0.14$ .

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**Table 1. Characteristics of dSphs**

Galaxies*	$\langle [\text{Fe}/\text{H}] \rangle^\dagger$	$M_*^\ddagger (M_\odot)$	$M_{1/2}^\S (M_\odot)$	$M_h^\P (M_\odot)$	$k^\parallel$	$\eta^{**}$
Fornax	-0.99	$1.9 \times 10^7$	$7.4 \times 10^7$	$2.8 \times 10^9$	0.30	14
Leo I	-1.43	$4.6 \times 10^6$	$2.2 \times 10^7$	$1.1 \times 10^9$	0.14	23
Leo II	-1.62	$1.4 \times 10^6$	$7.3 \times 10^6$	$5.4 \times 10^8$	0.15	36
Sculptor	-1.68	$1.2 \times 10^6$	$2.3 \times 10^7$	$4.8 \times 10^8$	0.13	38
Draco	-1.93	$9.1 \times 10^5$	$2.1 \times 10^7$	$3.8 \times 10^8$	0.052	40
Sextans	-1.93	$8.5 \times 10^5$	$3.5 \times 10^7$	$3.6 \times 10^8$	0.056	41
Ursa Minor	-2.13	$5.6 \times 10^5$	$5.6 \times 10^7$	$2.8 \times 10^8$	0.023	47
Canes Venatici I	-1.98	$3.0 \times 10^5$	$2.8 \times 10^7$	$2.0 \times 10^8$	0.10	65
Hercules	-2.41	$3.7 \times 10^4$	$7.5 \times 10^6$	$5.7 \times 10^7$	0.078	147
Ursa Major I	-2.18	$1.9 \times 10^4$	$1.3 \times 10^7$	$4.4 \times 10^7$	0.31	225
Leo IV	-2.54	$8.5 \times 10^3$	$1.1 \times 10^6$	$2.4 \times 10^7$	0.13	277
Canes Venatici II	-2.21	$8.0 \times 10^3$	$1.4 \times 10^6$	$2.9 \times 10^7$	0.48	351
Ursa Major II	-2.47	$5.4 \times 10^3$	$7.9 \times 10^6$	$2.0 \times 10^7$	0.24	357
Coma Berenices	-2.60	$4.8 \times 10^3$	$2.0 \times 10^6$	$1.8 \times 10^7$	0.16	354

\* Dwarf spheroidal galaxies in descending order of  $M_*$ .

<sup>†</sup> Mean metallicities taken from [1].

<sup>‡</sup> Stellar masses taken from [2] for the seven most massive dSphs and from [3] for the rest.

<sup>§</sup> Total dynamic masses within half-light radii taken from [11].

<sup>¶</sup> Estimated total masses of dark matter halos.

<sup>||</sup> Estimated relative rates of SNe Ia to CCSNe.

<sup>\*\*</sup> Estimated efficiencies of SN-driven outflows.